# Sophisticated Voting in an Approval Primary

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**Abstract**

Approval voting is promoted because no voter needs to abandon a favorite. St. Louis uses an approval primary to choose the two candidates for a general election run-off, and 21 of its 32 contested primaries had three or more candidates. I show that when three candidates run and voters know who would win each run-off, sophisticated voting turns the primary into a referendum on whether the Condorcet winner reaches the run-off, and the Condorcet winner wins. Under these assumptions, no other candidate has a reason to run in order to win. The crowded fields therefore show candidates acting as if they do not know who the Condorcet winner is. An approval primary followed by a run-off does not keep the promise of approval voting. Some voters do best by withholding approval from their favorite, and rival camps can collude to shut out the Condorcet winner at no cost when enough voters are sincere.

*Keywords:* approval voting, run-off elections, sophisticated voting, strategic voting, dominance solvability, candidate entry, Condorcet winner

## 1 Introduction

In 2020, St. Louis voters adopted Proposition D, which replaced partisan primaries for municipal elections with an approval primary that chooses the two candidates for a general election run-off. I call this electoral rule approval voting with a run-off (AVR). The campaign for Proposition D said voters could support grassroots candidates "without fear of 'splitting the vote' or 'spoiling the election'" (Center for Election Science, 2020). Crowded fields have been common under the rule, with three or more candidates in 21 of its 32 contested primaries.

In this paper, I show that with three candidates and voters who know who would win each run-off, sophisticated voting turns the approval primary into a referendum on whether

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the Condorcet winner (the candidate who beats every rival head to head) gets a place in the run-off. Because a majority prefers the Condorcet winner to each rival, the Condorcet winner wins regardless of who else runs, so no other candidate has a reason to enter.

The main contribution of this paper is to show that crowded fields are evidence about what candidates know. This has implications for what voters know, since voters are unlikely to know more about a race than the candidates do. A reader who takes the theory for approval voting with a run-off at face value might conclude that these candidates act irrationally. However, the result assumes that voters know who would win each run-off, and the crowded fields show candidates acting as if they do not know who the Condorcet winner is. Such behavior is also consistent with not expecting sophisticated voting from their electorate. That expectation has a basis, because a rival camp can shut the Condorcet winner out of the run-off at no cost when enough of that candidate's supporters vote sincerely, which I prove in Section 6. Ballot records from the 2025 primaries support that expectation, since at least 43 percent of the ballots in each primary with three candidates took forms that sophisticated voting rules out.

The second contribution is the theory behind this reading. Fishburn and Brams (1981) compared approval voting with the plurality run-off, and Delemazure et al. (2022) study AVR with sincere ballots. Neither study asks which ballots survive elimination of dominated strategies when the two stages are combined into a single electoral rule, which is the precise electoral context in St. Louis. Because these ballots are dominant, sophisticated voting needs no beliefs about others' votes, while strategic voting usually means equilibrium play under such beliefs (Myerson and Weber, 1993; Cox, 1997).

## 2 Approval primaries in St. Louis

Most contested primaries under AVR in St. Louis have had three or more candidates. Of 32 contested primaries, 16 had three candidates, 11 had two, and 5 had four to six. A primary with two candidates eliminates no one, because both advance to the run-off, and the run-off then elects the majority winner regardless of the primary ballots. The two-candidate case is trivial in theory, and my analysis covers 27 of the 32 contested primaries, the 16 with three candidates and the 11 with two. Among the 21 primaries with three or more candidates, 16 had three. Table 1 lists, for each election from 2021 to 2025, the number of contested primaries and the contests in which three or more candidates ran.

Table 1: Contested approval primaries in St. Louis, 2021 to 2025

| Election | Contested primaries | Three or more candidates | Contests with three or more candidates |
|---|---|---|---|
| March 2021 | 14 | 8 | Mayor (4); Wards 1, 4, 12, 15, 17, 21 (6), and 27 |
| September 2022 | 1 | 0 | None |
| March 2023 | 13 | 9 | Wards 1, 2, 4, 7, 8, 9, 12 (5), 13, and 14 (4) |
| March 2025 | 4 | 4 | Mayor (4); comptroller; Wards 3 and 11 |
| Total | 32 | 21 | |

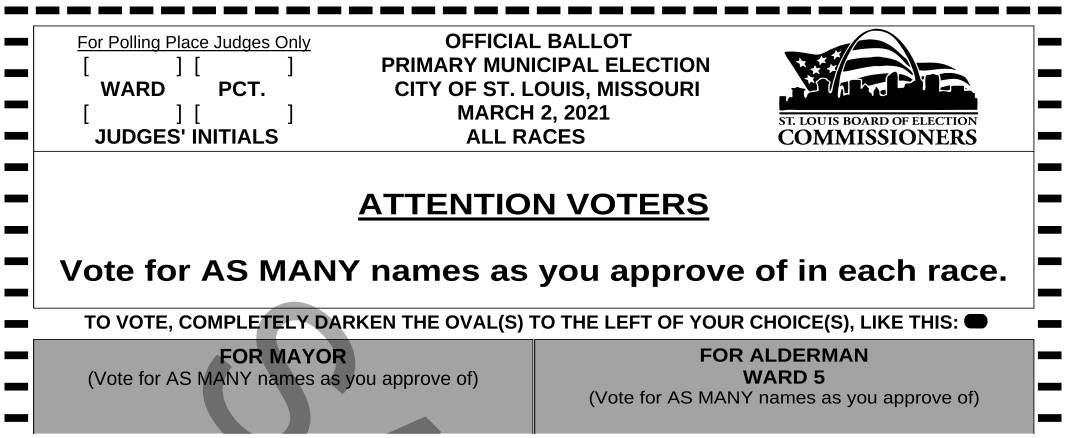
*Notes:* A primary is contested when two or more candidates ran. Three candidates ran unless a number in parentheses gives more. Wards are numbered on the 28-ward map in 2021 and on the 14-ward map from 2023, and Wards 4 and 12 in 2021 filled unexpired terms. The September 2022 primary was a special election for president of the Board of Aldermen. Source: St. Louis Board of Election Commissioners results as tabulated by approval.vote and Ballotpedia.

Crowded fields were also common before Proposition D. Until 2021, the city chose nominees in partisan primaries, and the Democratic primary usually decided who would hold the office. In the 46 contested Democratic primaries from 2013 to 2019, 24 had two candidates, 12 had three, and 10 had four to seven, so 22 had three or more. The largest field was the seven-candidate mayoral primary of 2017. Contested primaries averaged 2.9 candidates both before and after the change, while the share with three or more candidates rose from 48 to 66 percent. Part of that rise comes from Ordinance 71736, which kept two-candidate contests out of the 2025 primary, and different offices were on the ballot in different years, so the counts do not show that the rule crowded the fields.[1]

Proposition D passed on November 3, 2020, by 87,305 votes to 40,897. Primaries are held in March and run-offs at the general election in April, and candidates appear without party labels.[2] Figure 1 shows the top of the sample ballot for the first primary held under the rule, on March 2, 2021. The ballot told voters to "Vote for AS MANY names as you approve of in each race."

[1]Sources: Ballotpedia, Urban Review STL, Wikipedia, and St. Louis Public Radio. The counts cover the Democratic primaries for mayor, comptroller, president of the Board of Aldermen, and alderman, including special elections, and leave out the other parties' primaries.

[2]The rule does not cover every city election. The city charter fills vacancies on the Board of Aldermen through partisan special elections, and on November 3, 2026, a charter amendment that would have ended this requirement received 56 percent of the vote in the unofficial count, short of the 60 percent that charter amendments need (Lippmann, 2026).


For Polling Place Judges Only
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WARD PCT.
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JUDGES' INITIALS

OFFICIAL BALLOT
PRIMARY MUNICIPAL ELECTION
CITY OF ST. LOUIS, MISSOURI
MARCH 2, 2021
ALL RACES

ST. LOUIS BOARD OF ELECTION COMMISSIONERS

ATTENTION VOTERS

Vote for AS MANY names as you approve of in each race.

TO VOTE, COMPLETELY DARKEN THE OVAL(S) TO THE LEFT OF YOUR CHOICE(S), LIKE THIS:

FOR MAYOR
(Vote for AS MANY names as you approve of)

FOR ALDERMAN
WARD 5
(Vote for AS MANY names as you approve of)

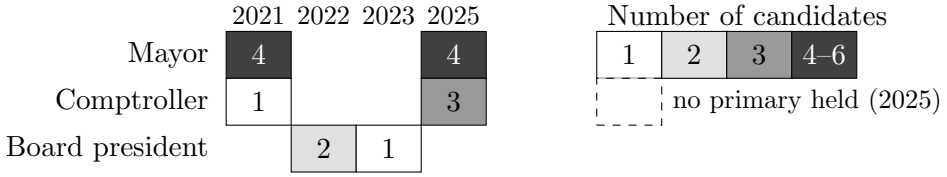
Figure 1: The top of the sample ballot for the primary of March 2, 2021, the first held under Proposition D. Source: St. Louis Board of Election Commissioners (2021).

Two later changes reduced the number of primaries. Voters adopted Proposition R in 2012, and it cut the Board of Aldermen from 28 wards to 14 from the 2023 elections. Ordinance 71736 lets contests with two or fewer candidates skip the primary, a provision first applied in 2025. Because of the ordinance, every primary held in 2025 had three or more candidates, and three of the four had exactly three.

Figure 2 shows the number of candidates in every contest held under the rule. The primaries with three or more candidates were spread across the city (Table 1). In 2021, they were the mayoral primary and the primaries in seven of the 28 wards. In 2023, they were the primaries in nine of the 14 wards.

Five primaries had more than three candidates, namely the mayoral primaries of 2021 and 2025, with four each. The 2021 primary in Ward 21 had six, and the 2023 primaries in Wards 12 and 14 had five and four, respectively. Only the mayoral primary had more than three candidates in two elections. Ward 21 did not survive the change to 14 wards, and Wards 12 and 14, like the other even-numbered wards on the new map, have not held a regular election since 2023.

**(a) Citywide offices**




**(b) Wards on the 28-ward map, 2021**

| Ward | 1 | 3 | 4 | 5 | 7 | 9 | 11 | 12 | 13 | 15 | 17 | 19 | 21 | 23 | 25 | 27 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2021 | 3 | 2 | 3 | 2 | 2 | 2 | 1 | 3 | 2 | 3 | 3 | 2 | 6 | 1 | 1 | 3 |

**(c) Wards on the 14-ward map, 2023 and 2025**

| Ward | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2023 | 3 | 3 | 1 | 3 | 2 | 2 | 3 | 3 | 3 | 2 | 2 | 5 | 3 | 4 |
| 2025 | 2 | | 3 | | 1 | | 2 | | 1 | | 3 | | 1 | |

Figure 2: The number of candidates in each contest held under AVR, 2021 to 2025. Darker cells had more candidates, and a dashed border marks a 2025 contest that skipped the primary because it had two or fewer candidates. Sources as in Table 1, plus Wikipedia and St. Louis Public Radio for three uncontested races.

# 3 The game with three candidates

Three candidates face $N$ voters, each with a strict ranking. I compare the candidates by majority rule.

**Definition 1** (Majority relation)**.** Candidate $x$ beats candidate $y$ if more than half of voters rank $x$ above $y$. The majority relation states for each pair of candidates which one beats the other. A Condorcet winner beats both other candidates, and a Condorcet loser loses to both.

I assume no pairwise ties. The majority relation depends only on the voters' rankings, so which candidate is the Condorcet winner does not change with the ballots that voters cast in either stage. The combined rule AVR elects a winner in two stages. In the approval primary, each voter approves any subset of the candidates, and the two candidates with the most approvals advance. Ties are broken by a fair lottery.[3] In the run-off, each voter votes for

[3]A voter may approve no candidate or every candidate, and public results show how often voters did each. In the mayoral primary of March 4, 2025, voters cast 34,982 ballots, and 37 voters approved none of the four candidates. The other 34,945 carried 48,908 approvals, or 1.4 per ballot, and 0.4 percent of them approved all four. In the comptroller primary on the same ballot, 1,315 ballots approved none of the three candidates. The other 33,667 carried 41,403 approvals, or 1.2 per ballot, and 2.8 percent of them approved all three. Sources: St. Louis Board of Election Commissioners results as tabulated by approval.vote and Ballotpedia.

one of the two candidates who advanced. The majority winner then takes office. Compare this to single-round approval voting (AV), where the approval round is the whole election, and the candidate with the most approvals wins.

An example shows AVR at work, and I return to it throughout the paper. In an electorate of 100 voters, L is a progressive Democrat, C is a moderate Democrat, and R is a Republican. The voters rank the candidates as follows, where $x \succ y$ means that $x$ is ranked above $y$:

$$\begin{aligned} &33 \text{ voters:} \quad \text{L} \succ \text{C} \succ \text{R}, \\ &40 \text{ voters:} \quad \text{C} \succ \text{L} \succ \text{R}, \\ &27 \text{ voters:} \quad \text{R} \succ \text{C} \succ \text{L}. \end{aligned}$$

The pairwise contests make C the Condorcet winner:

$$\text{C beats L, 67 to 33,} \qquad \text{C beats R, 73 to 27,} \qquad \text{L beats R, 73 to 27.}$$

Preferences are single-peaked on the order L, C, R, and C is also the median candidate (Black, 1948). Figure 3 shows how each pairwise contest splits the voters at one point on this order, so that the side holding the median voter wins and C beats both rivals.

Figure 4 evaluates the approval primary and run-off on this electorate when everyone votes sincerely, with L's and R's voters approving only their favorite and 10 of C's 40 voters also approving L. L and C advance with 43 and 40 approvals, R is out with 27, and C wins the run-off 67 to 33 because R's voters prefer C to L. Approval voting alone would have ended after the approval primary and elected L, the candidate with the most approvals.

In a run-off between two candidates, voting for one's preferred candidate is weakly dominant, and I assume that every voter does so. I also assume that the run-off electorate is the primary electorate. A run-off between $x$ and $y$ then elects $W(x, y)$, the majority winner of the pair. Removing dominated run-off votes is the first round of eliminating dominated strategies, and the analysis of the primary below is the second.

Voters know the majority relation, and hence $W$ for every pair. Voters know nothing about how others will vote in the primary. Once the other voters' ballots are fixed, a voter's ballot induces a lottery over winners, because ties in the primary are broken by a fair lottery. I compare ballots by these lotteries.

**Definition 2** (Dominance)**.** A lottery $P$ over the candidates first-order stochastically dominates a lottery $Q$ for a voter if, for every candidate $x$, $P$ gives at least as high a probability as $Q$ to the election of $x$ or of a candidate the voter ranks above $x$. Ballot $B$ weakly dominates ballot $B'$ for a voter if, for every profile of the other voters' ballots, the lottery over winners

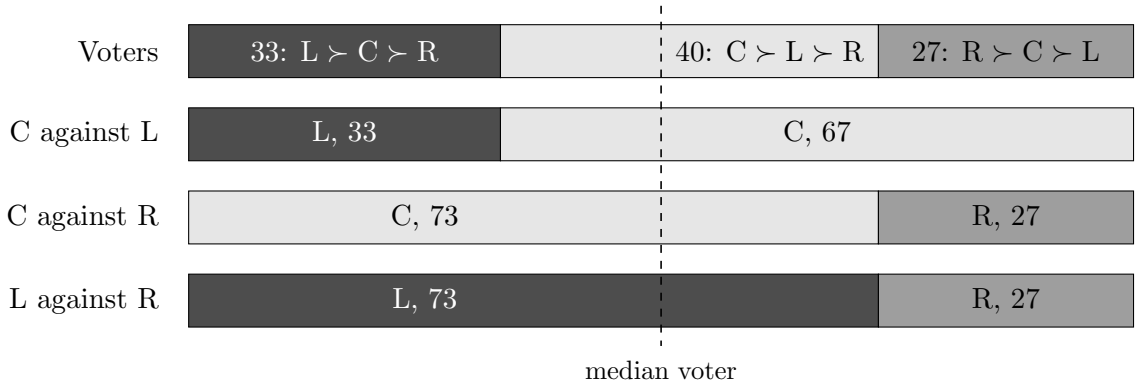



Figure 3: The electorate of the example. The top bar orders the 100 voters by their favorite candidate, from left to right, and each lower bar splits the same voters by the candidate they prefer in one pairwise contest. Because preferences are single-peaked, every contest splits the voters at a single point, and the side that holds the median voter (dashed line) wins. C therefore beats both rivals, and L beats R.

induced by ballot $B$ first-order stochastically dominates the lottery induced by $B'$, and the two lotteries differ for at least one profile. Ballot $B$ strictly dominates $B'$ if the two lotteries also differ for every profile. A ballot is dominant if it weakly dominates every other ballot.

I define strict and weak dominance this way so that the results hold for any von Neumann–Morgenstern utility function consistent with the voter's ranking. A lottery first-order stochastically dominates another exactly when every such utility function gives it at least as high an expected utility (Hadar and Russell, 1969; Hanoch and Levy, 1969).

I distinguish three kinds of voting. *Sincere voters* approve a top segment of their ranking. With three candidates, this means their favorite and possibly their second choice (Brams and Fishburn, 1978). *Sophisticated voters* cast only ballots that survive the elimination of dominated strategies, which here requires knowledge of the majority relation and no beliefs about how others will vote (Farquharson, 1969; McKelvey and Niemi, 1978). *Strategic voters* choose a best response to their beliefs about how others will vote (Myerson and Weber, 1993; Cox, 1997). The two stages of AVR treat these kinds of voting differently. Under approval voting alone with three candidates, every undominated ballot approves the voter's favorite and never the voter's last choice, so every undominated ballot is sincere (Brams and Fishburn, 1978). However, a voter still has two undominated ballots, and choosing between them is strategic. A run-off between two candidates leaves each voter a single undominated vote, which is sincere. In Section 4, I show that AVR also leaves each voter a single dominant ballot, which makes strategic voting unnecessary, despite that ballot being insincere for the voters who rank the Condorcet winner second.[4]

[4]Under AV and under AVR alike, a ballot that approves no one and a ballot that approves every candidate

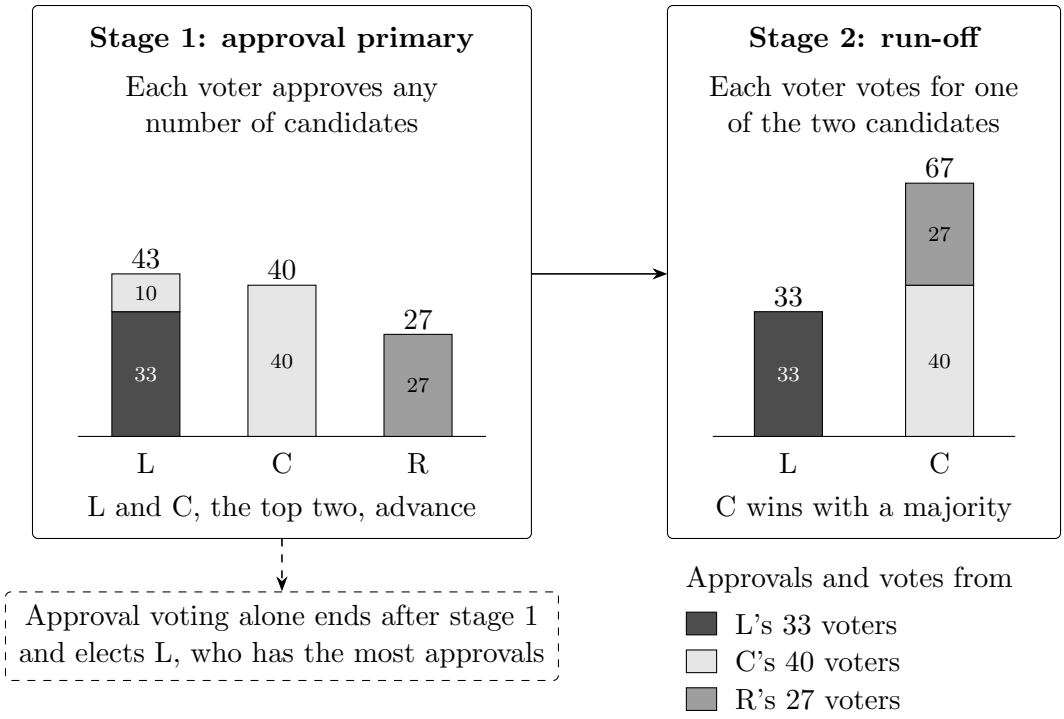



Figure 4: How AVR elects a winner in the example election when every voter votes sincerely. Each bar shows a candidate's approvals or votes, divided by the voters who cast them. The approval primary sends L and C to the run-off, and C wins the run-off because R's voters prefer C to L. Approval voting alone would end after stage 1 and elect L.

I assume throughout that a Condorcet winner exists. I write $c$ for it and $a$ and $b$ for the rivals, labeled so that $a$ beats $b$. Then $b$ is the Condorcet loser, and $a$ is the candidate who wins if $c$ is kept out of the run-off. In the example:

> Moderate Democrat C plays the role of the Condorcet winner $c$.
>
> Progressive Democrat L plays the role of $a$, who wins the run-off held without $c$.
>
> Republican R plays the role of $b$, the Condorcet loser.

Because voters know the majority relation, they can predict the winner of every possible run-off, and that prediction leaves the primary with a single question to decide.

If $c$ gets one of the two places in the run-off, $c$ wins it, since $c$ beats both rivals. If $c$ does not, the run-off pairs $a$ with $b$, and $a$ wins. The primary therefore affects the outcome

are dominated, and they have the same effect on the count, since neither changes the difference between any two candidates' totals. They differ in what a voter who chose to turn out intends, because the first rejects the whole field and the second accepts all of it. A none-of-the-above option would let a voter record the first intent on an approval ballot, and it would change the outcome only if the rule let that option win or force a new election.

only through whether the Condorcet winner gets one of the two places in the run-off, and Figure 5 shows the two branches. In the example, the question is whether C gets one of the two places in the run-off.

# 4 The referendum

With three candidates, sophisticated voting makes the approval primary a referendum on whether the Condorcet winner gets one of the two places in the run-off. I call the ballot that approves the Condorcet winner $c$ alone the Yes Ballot and the ballot that approves $a$ and $b$ together the No Ballot. The lemma below shows that every voter's dominant ballot is one of the two.

**Lemma 1** (The referendum)**.** *Suppose a Condorcet winner exists. Under AVR with three candidates and a known majority relation, the game is dominance solvable in two rounds. The first round removes every vote for the less preferred finalist of a run-off, and the second leaves every voter a single primary ballot. A voter who ranks c above a votes yes by approving c alone, and a voter who ranks a above c votes no by approving a and b. When every voter casts that ballot, c is approved on more than half of the ballots, a and b each on the rest, and c wins.*

*Proof.* A strategy that votes for the less preferred finalist of some run-off is weakly dominated by the strategy that differs only in voting for the preferred finalist there, and removing such strategies is the first round. In the game that remains, if $c$ gets one of the two places in the run-off, $c$ wins it, and if $c$ does not, the run-off pairs $a$ with $b$ and $a$ wins. The primary therefore decides one question, whether $c$ gets a place, and a ballot affects the outcome only through $c$'s chance of getting one. Under fair tie-breaking, that chance is nondecreasing in $c$'s total and nonincreasing in each rival's, so approving $c$ can only raise it and approving $a$ or $b$ can only lower it, for every profile of the others' ballots. A voter who prefers $c$ to $a$ therefore does best with the Yes Ballot $\{c\}$, and one who prefers $a$ to $c$ does best with the No Ballot $\{a, b\}$. Every other ballot does strictly worse against some profile, one in which it changes whether $c$ gets a place, and removing these ballots is the second round. Because $c$ beats $a$, the voters who vote yes are a majority, and $c$ finishes first and wins the run-off. □

The lemma is the approval-ballot analogue of sophisticated voting on an agenda (Farquharson, 1969; McKelvey and Niemi, 1978). With three candidates, AVR is a two-stage agenda in which the primary picks one of three pairs and the run-off chooses within it. Because $c$ wins both pairs that include $c$, sophisticated voters treat the primary as a vote

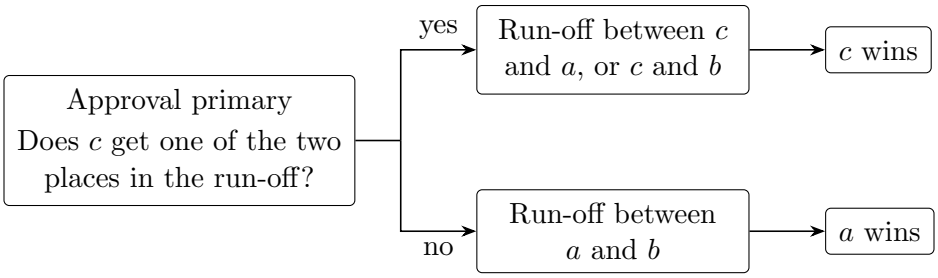



Figure 5: The question that the approval primary decides. With three candidates and a known majority relation, the Condorcet winner $c$ wins any run-off it reaches, and $a$, the candidate who beats $b$, wins the run-off held without $c$. These roles depend only on the voters' rankings, regardless of the ballots they cast.

on whether $c$ gets a place. The outcome follows from the analysis by Bag et al. (2009) of procedures that eliminate one candidate per round.

I evaluate the two elections backward through the combined electoral rule, using the first round of elimination for the run-off at the general election and the second for the primary. Because these two rounds leave each voter a single strategy, the game is dominance solvable in the sense of Moulin (1979).

The logic of the second round follows Figure 5. Approving $c$ can only raise the chance that $c$ gets a place in the run-off, and approving $a$ or $b$ can only lower it, no matter what the other voters do. The Yes Ballot is best for a voter who wants $c$ to get a place, and the No Ballot is best for a voter who does not. Table 2 lists the dominant ballot for each of the six rankings, grouped by the answer to the referendum.

Four of the six dominant ballots are sincere. The other two belong to the voters who rank the Condorcet winner second, and Corollary 1 describes them.

**Corollary 1** (The price of sincerity)**.** *The voters whose second choice is the Condorcet winner must vote insincerely. Those whose favorite would win without the Condorcet winner vote no and approve their last choice, and those whose favorite is the Condorcet loser vote yes and withhold approval from their favorite.*

Figure 6 in Section 6 shows why the voters who rank the Condorcet winner second split between the two answers.

Dominant play also fixes the pattern of the totals. When every voter casts the ballot in Table 2 for that voter's ranking, no voter needs a belief about how the others will vote, and each ballot approves either $c$ alone or $a$ and $b$ together. The number of Yes Ballots is then $c$'s total, and the number of No Ballots is the total of each rival.

Table 2: Dominant primary ballots under AVR with three candidates, grouped by the answer to the referendum

| Ranking | Answer | Dominant ballot | Sincerity |
|---|---|---|---|
| $c$ over $a$ over $b$ | yes | $\{c\}$ | sincere |
| $c$ over $b$ over $a$ | yes | $\{c\}$ | sincere |
| $b$ over $c$ over $a$ | yes | $\{c\}$ | withholds approval from the favorite |
| $a$ over $b$ over $c$ | no | $\{a, b\}$ | sincere |
| $b$ over $a$ over $c$ | no | $\{a, b\}$ | sincere |
| $a$ over $c$ over $b$ | no | $\{a, b\}$ | approves the last choice |

**Corollary 2** (The referendum signature)**.** *Under dominant play, only the Yes Ballot and the No Ballot occur. The two rivals receive identical totals, and c's total plus either rival's equals the number of ballots.*

In the example of Section 3, the 67 voters who prefer C to L vote yes, and the 33 who prefer L to C vote no. The totals are C 67, L 33, and R 33, and C gets a place in the run-off and wins it against either rival. R's voters answer yes by approving C and withholding approval from their favorite, and L's voters answer no by approving R, their last choice.

Approval ballots also give every voter a dominant way to answer the question of whether $c$ gets a place in the run-off, which plurality ballots do not. A plurality top-two primary elects $c$ as well when voters are sophisticated. However, voters who prefer $a$ to $c$ have no dominant vote there, since voting for $a$ and voting for $b$ are both undominated (compare Niou, 2001). Under AVR they approve both, which is the No Ballot, and the signature of Corollary 2 follows.

The lemma does not conflict with the theorem of Gibbard (1973), which says that a voting scheme with three or more possible outcomes gives every voter a dominant strategy for every preference only if it is dictatorial. The lemma's dominance holds only in the game that remains once the dominated run-off votes are removed, and that game depends on the majority relation, which depends on the preferences of all voters. A voter's dominant ballot depends on what others prefer as well as on what the voter prefers.

# 5 What crowded primaries say about candidates

Lemma 1 concerns voters and takes the field of candidates as given. It still has an implication for who has a reason to run, because it says who wins.

**Corollary 3** (Who can win)**.** *Under the assumptions of Lemma 1, a candidate who is not the Condorcet winner cannot win. If a third candidate who is not the Condorcet winner*

*enters a race between two candidates, the winner does not change.*

With two candidates, both advance, and the run-off elects the majority winner of the pair. If a third candidate enters and is not the Condorcet winner of the three, the Condorcet winner is the candidate who won the pair, since that candidate beats the other one, and Lemma 1 says that this candidate wins. Under these assumptions, only the Condorcet winner has a reason to run in order to win, and if running has any cost, every other candidate who runs only to win stays out.

The St. Louis primaries do not fit this prediction. Every contested primary had at least one candidate who could not win under these assumptions, and the 21 primaries with three or more candidates in Table 1 had two or more. Despite this, the candidates' behavior is still consistent with the model's assumptions.

Corollary 3 depends on the knowledge provided to voters in Lemma 1, which is knowledge of who would win each possible run-off. A candidate with that knowledge who is not the Condorcet winner knows that running cannot win the office. Crowded fields therefore show candidates acting as if they lack that knowledge. Each candidate who runs against the Condorcet winner is unsure who the Condorcet winner is *or* expects voters not to vote sophisticatedly.[5] Some candidates may also run for reasons that the model leaves out, such as raising an issue (Canon, 1990) or building name recognition for a later race (Haime et al., 2022).

A candidate who expects some sincere voting has a reason to run, because when the Condorcet winner's supporters vote sincerely, a rival camp can shut the Condorcet winner out of the run-off at no cost (Proposition 1 in Section 6), and a candidate other than the Condorcet winner then wins. Ballot records show that this expectation is reasonable. For each 2025 primary, approval.vote summarizes the cast vote records, and its figures give the number of ballots that approved each candidate alone, each pair, and all three. Under dominant play, every ballot is either a Yes Ballot or a No Ballot. In the three 2025 primaries with three candidates, ballots of other types made up 55 percent of the ballots for comptroller, 52 percent in Ward 3, and 43 percent in Ward 11, even when I choose the Condorcet winner that fits the ballots best.[6] Between 77 and 89 percent of the ballots in each of these primaries approved a single candidate, which is how a sincere voter who likes only a favorite votes.

[5]Recall from Section 3 that strategic behavior is unnecessary for analyzing AVR when voters know who would win each run-off, because each voter then has a single dominant ballot.

[6]For each primary and each choice of the Condorcet winner, I count the ballots that approve that candidate alone or the other two candidates together, and I keep the largest count. That count is 15,127 of the 33,667 ballots that approved anyone for comptroller, 777 of 1,624 in Ward 3, and 656 of 1,152 in Ward 11. The 2025 mayoral primary had four candidates and lies outside the model. Source: St. Louis Board of Election Commissioners cast vote records as summarized by approval.vote.

How the fields changed from one election to the next can show whether candidates learn who would win. Each election reveals part of the majority relation. A run-off shows which of its two candidates a majority prefers, and the primary totals show how widely each candidate is approved. A candidate who loses learns that a majority preferred someone else, and potential candidates who watch learn the same. If candidates learn in this way, crowded fields should become less common in later elections for the same office, unless something changes the majority relation, such as new ward boundaries or an incumbent's record in office.

Figure 2 shows these changes. The city redrew its wards for 2023, so of the 2021 primaries with three or more candidates, only the mayoral primary can be followed to a later election, and it had four candidates in both 2021 and 2025. On the new map, only the odd-numbered wards held elections in both 2023 and 2025. None of the four odd-numbered wards that had three candidates in 2023 had more than two in 2025, while the two wards with three candidates in 2025 had at most two in 2023. With so few wards, the comparison can only suggest that candidates learn.

Every 2025 primary with three or more candidates included an incumbent, and incumbency can shape both who runs (Jacobson and Kernell, 1981; Cox and Katz, 1996) and how voters judge the majority relation (Mann and Wolfinger, 1980). An incumbent who runs again has already won the office once, which is evidence that a majority preferred the incumbent to the alternatives at the time. If preferences had not changed, that evidence would mark the incumbent as a likely Condorcet winner and would keep out challengers who run only to win. However, a record in office gives voters new information, and a challenger who expects that information to have turned a majority against the incumbent has a reason to run. Voters also know more about an incumbent than about a newcomer, so the pairs that include the incumbent may be the easiest part of the majority relation to know. When voters believe that the incumbent is the Condorcet winner, the primary becomes a referendum on the incumbent, much as general elections are often read as referendums on incumbents (Key, 1966; Fiorina, 1981). The incumbent mayor and comptroller both lost their run-offs, which fits challengers who doubted that the incumbent was the Condorcet winner.[7]

# 6 The open raid

As in Lemma 1, I assume throughout this section that the electoral rule is AVR with three candidates and that voters know the majority relation.

[7]Sources: St. Louis Board of Election Commissioners results as tabulated by approval.vote and Ballotpedia, and St. Louis Public Radio for incumbents.

Studies of primaries call it raiding when supporters of one party cross over into another party's open primary to vote for the candidate their own party would most like to face (Alvarez and Nagler, 2002; Chen and Yang, 2002; Cho and Kang, 2015). A crossover raid has a cost, because a raider gives up a vote in the raider's own party's contest. AVR has no party contests, and every voter casts the same nonpartisan ballot. A camp of sophisticated voters can therefore approve a weak rival without withdrawing approval from its own candidate, and when other voters vote sincerely, the camp can shut the Condorcet winner out of the run-off at no cost. I call this an open raid. The name refers to a primary that is open to every voter, and an open raid differs from a crossover raid in an open partisan primary because the raiders give up nothing. The open raid uses the No Ballot, and the voters who rank the Condorcet winner second show why that ballot can approve a voter's last choice.

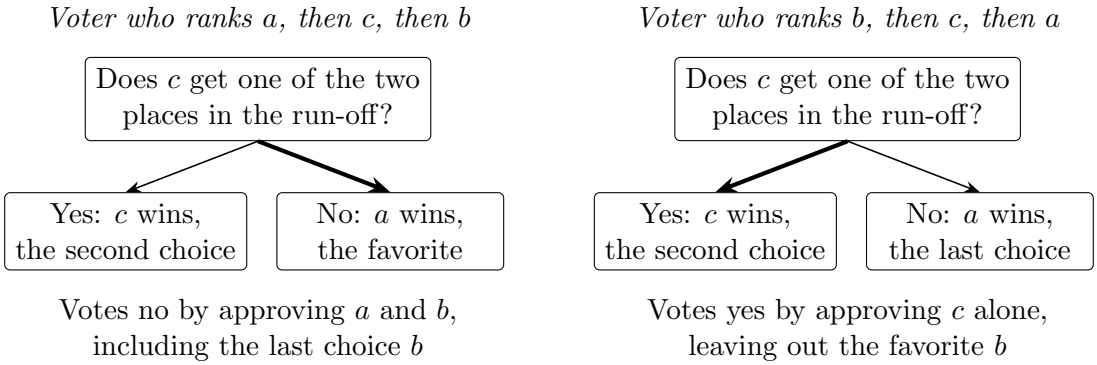



Figure 6: Why the two kinds of voter who rank the Condorcet winner second answer the referendum differently. The thick arrow marks the answer each voter prefers. The Condorcet loser $b$ loses every run-off, and approving $b$ therefore matters only through its effect on whether $c$ gets a place.

Figure 6 shows why the two kinds of voter who rank the Condorcet winner second answer the referendum differently. Both get their second choice if $c$ gets a place in the run-off. If $c$ is shut out, $a$ wins, which is the favorite of a voter who ranks $a$ first and the last choice of a voter who ranks $b$ first. A voter who ranks $a$, then $c$, then $b$ therefore votes no, and the No Ballot approves $b$, the voter's last choice. Approving $b$ costs this voter nothing, because $b$ loses every run-off and can only help push $c$ out of the top two. A voter who ranks $b$, then $c$, then $a$ votes yes and withholds approval from $b$, the voter's favorite, because approving $b$ could only help shut $c$ out and elect $a$.

Both of these ballots are dominated under approval voting alone, where every undominated ballot approves the voter's favorite and never the voter's last choice (Brams and Fishburn, 1978). Merrill and Nagel (1987) found that multistage procedures can make strictly insincere approval ballots optimal, and Corollary 1 says which voters must cast them under

AVR. Proposition D offered the approval ballot as a way for voters to support every candidate they like. For sophisticated voters under AVR, the ballot instead records the answer to a single yes-or-no question—whether the Condorcet winner gets a place in the run-off. Approving more than one candidate then has a single use, which is to vote no, because the No Ballot is the only dominant ballot that approves more than one candidate.

The No Ballot of a voter who ranks $a$ first is also the ballot of an open raid on the Condorcet winner. Lemma 1 assumes that every voter answers the referendum with a dominant ballot, while actual electorates mix sincere voters with sophisticated ones. Table 3 shows an example election in which the voters who prefer $a$ to $c$ cast the No Ballot, everyone else votes sincerely, and the no side wins at no cost to the raiders.

Table 3: An example election with an open raid

| | | Ballot cast | | |
|---|---|---|---|---|
| Voters | Ranking | (1) Sincere | (2) Raid with approval ballots | (3) Raid with plurality ballots |
| 33 | L over C over R | L | L and R | R |
| 30 | C over L over R | C | C | C |
| 10 | C over L over R | C and L | C and L | C |
| 27 | R over C over L | R | R | R |
| Total for L | | 43 | 43 | 0 |
| Total for C | | 40 | 40 | 40 |
| Total for R | | 27 | 60 | 60 |
| Run-off | | C against L | L against R | C against R |
| Winner | | C | L | C |

*Notes:* The 100 voters have the preferences of the example in Section 3, and C is the Condorcet winner. In column (2), L's voters add R to their ballots, which lifts R above C without taking an approval from L. In column (3), the same raid with one vote per voter costs L all of its votes.

The electorate has 100 voters with the preferences of the example in Section 3. With sincere ballots, in column (1), L's voters approve only L, R's voters approve only R, and 10 of C's 40 voters also approve L, as in Figure 4. L and C advance, and C wins the run-off against L. In column (2), L's voters raid by adding R to their ballots. R's total rises from 27 to 60 while L keeps its 43 approvals, C finishes third, and L wins the run-off against R.

The raid is costless because an approval for R takes nothing from L, and R cannot win a run-off against L. Column (3) shows why the same raid fails in a plurality top-two primary, where L's voters would have to vote for R instead of L. L then falls to zero, R and C advance, and C wins the run-off.

Figure 7 shows when such a raid succeeds. As in Figure 4, each bar divides a candidate's approvals by the voters who cast them, and C is shut out only when both rivals rise above the dashed line at C's total. In panel (b), the raid succeeds because 10 of C's voters also approved L, which lifted L above C. In panel (c), C's voters approve C alone, L falls to 33, and C finishes second and wins the run-off against R.

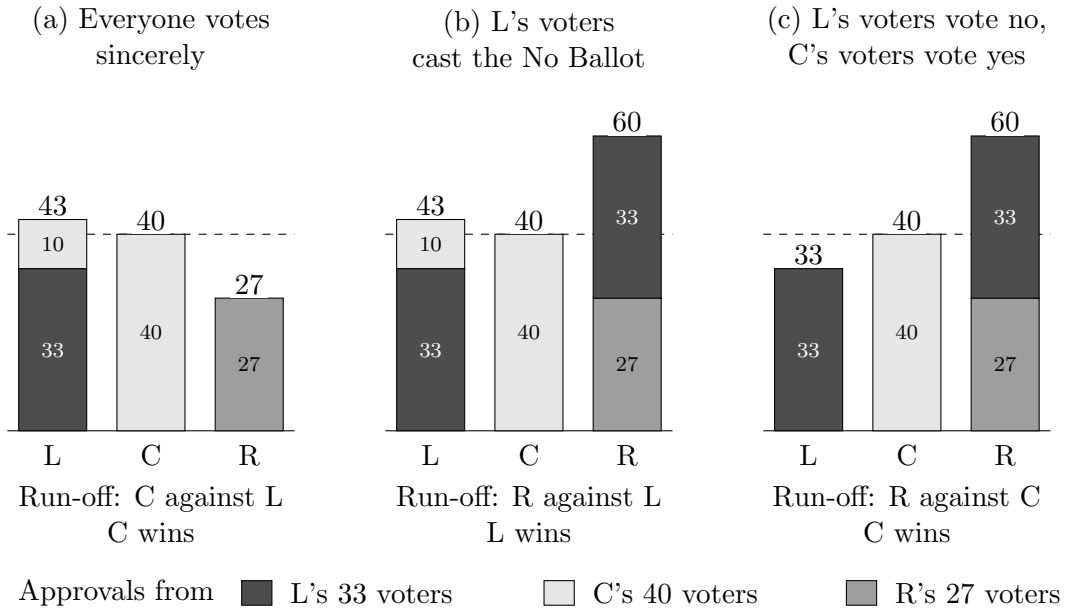



Figure 7: When the raid in Table 3 succeeds. Each bar shows a candidate's approvals, divided by the voters who cast them, and the dashed line marks C's total. C is shut out of the run-off only in panel (b), where both rivals rise above the line.

Figure 8 shows how much help the raid needs in this election. The figure holds R's voters at their sincere ballots and varies two numbers, the number of L's voters who add R and the number of C's voters who also approve L. C is shut out only in the shaded region, where at least 14 of L's voters add R and at least 8 of C's voters approve L. With fewer raiders, R stays below C's 40 approvals, and with fewer of C's voters approving L, L falls below C. In this election, the raid therefore needs help from some of C's own voters. The three panels of Figure 7 are the points (a), (b), and (c).

Proposition 1 states the general case. Part (i) says that the yes side wins whenever all of its voters cast the Yes Ballot, and part (ii) gives the condition under which a raid succeeds when they do not.

**Proposition 1** (The open raid)**.** *Under AVR with three candidates and a known majority relation, the following hold.*

*(i) If every voter who prefers c to a casts the Yes Ballot, then c wins, no matter what the*

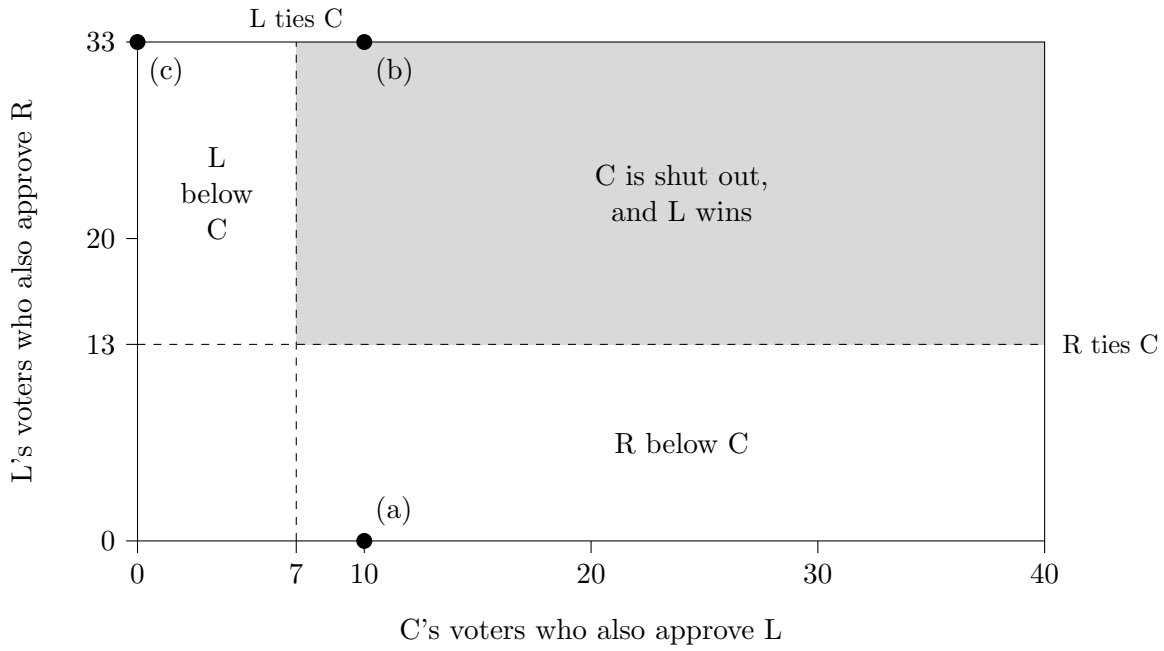



Figure 8: When the raid in Table 3 shuts C out of the run-off. R's 27 voters approve R alone throughout. In the shaded region, R and L both finish above C's 40 approvals, and C is shut out. Elsewhere C gets a place in the run-off and wins. On a dashed line a rival ties C, and a fair lottery decides. Points (a), (b), and (c) are the three panels of Figure 7.

*other voters do.*

*(ii) Suppose instead that the voters who prefer $a$ to $c$ make up a share $\nu < 1/2$ of the electorate and cast the No Ballot, while every other voter votes sincerely. Let $s_c$, $s_a$, and $s_b$ be the shares of the electorate made up of sincere voters who approve $c$, $a$, and $b$. Then $c$ is kept out of the run-off, and $a$ wins, if $s_c$ is below both $\nu + s_a$ and $\nu + s_b$.*

*Proof.* In part (i), $a$ and $b$ can be approved only by the voters who prefer $a$ to $c$, a minority, while $c$ is approved by a majority, so $c$ finishes first and wins the run-off. In part (ii), the totals are $s_c$ for $c$, $\nu + s_a$ for $a$, and $\nu + s_b$ for $b$, and $c$ is excluded when both rivals finish above $c$. The run-off then pairs $a$ with $b$, and $a$ wins. □

The raiders' approvals count toward both rivals and never toward $c$, which is why $\nu$ appears in both rivals' totals and why, in Figure 7, $c$ is shut out only when both rival bars rise above the dashed line. A sincere ballot from a voter who prefers $c$ to $a$ helps the raid whenever it approves a rival of $c$, because that approval helps the rival pass $c$. I call such a ballot a Partial No Ballot.

The raid costs the raiders nothing, because the No Ballot is already their dominant ballot and approving $b$ takes nothing from $a$. For the raiders, the open raid is a free lunch, and the

Condorcet winner's sincere supporters pay for it. An open raid also differs from a raid in a plurality top-two primary, as column (3) of Table 3 shows. In the example, a Democratic primary would have nominated C, 40 to 33. Under AV the same raid backfires since R is approved by 60 of the 100 voters and wins outright.

Figure 9 collects the conditions under which a costless raid is feasible. The approval ballot makes the raid costless because the raiders can approve $b$ without withdrawing approval from $a$. The run-off protects the raiders because $b$ cannot win a run-off against $a$. The raiders must prefer $a$ (who wins the run-off held without $c$) to $c$, and the raid succeeds only when both rivals, $a$ and $b$, finish above $c$. When every voter who prefers $c$ to $a$ approves $c$ alone, $c$ finishes above both rivals no matter what the raiders do.

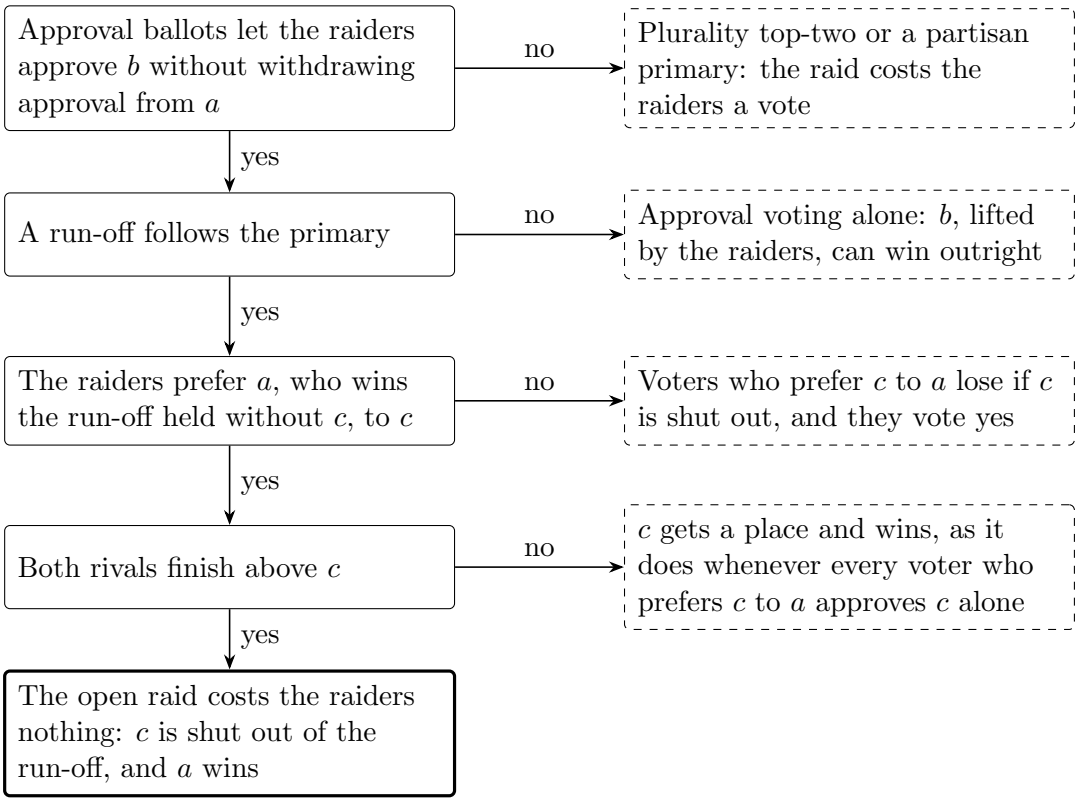



Figure 9: When an open raid on the Condorcet winner $c$ is feasible. The raiders cast the No Ballot $\{a, b\}$. The raid shuts $c$ out of the run-off only when every condition on the left holds, and each box on the right names what stops it.

# 7 Discussion

How sincere, sophisticated, and strategic behaviors serve a voter depends on the electoral rule. In the partisan primaries that St. Louis used before Proposition D, a voter whose

favorite trails has a reason to vote strategically for a stronger candidate, because a crowded field splits the vote (Cox, 1997). Under approval voting alone with three candidates, every undominated ballot is sincere, so no voter needs to abandon a favorite (Brams and Fishburn, 1978). AVR was adopted without a comparable theory of how voters would use an approval ballot when a run-off follows, and the campaign for it promised the benefits of approval voting (Center for Election Science, 2020). I have shown that an approval primary with a second stage does not preserve these benefits, because the run-off gives some voters an incentive to vote insincerely and gives a camp of sophisticated voters a way to manipulate the outcome at no cost.

The crowded fields of the St. Louis approval primaries are consistent with rational candidates who do not know who the Condorcet winner is. When three candidates run and voters know who would win each possible run-off, two rounds of eliminating dominated strategies turn the approval primary into a referendum on whether the Condorcet winner gets one of the two places in the run-off, and the Condorcet winner wins. Under these assumptions, a candidate with the same knowledge would run in order to win only if that candidate were the Condorcet winner. The 21 primaries with three or more candidates therefore show candidates who doubt who the Condorcet winner is or who expect some voters to vote sincerely. Some may also run for reasons the model leaves out. Earlier work could not draw this inference, because it took the ballots cast under AVR as sincere.

The expectation that some voters will vote sincerely has a basis in AVR. Voters who rank the Condorcet winner second lose the guarantee of a sincere ballot that approval voting alone gives them, because their dominant ballot either withholds approval from their favorite or approves their last choice. When enough of the Condorcet winner's supporters vote sincerely, a sophisticated minority can shut the Condorcet winner out of the run-off at no cost, which gives a candidate who is not the Condorcet winner a reason to expect a win. The open raid needs both stages of AVR, and it fails whenever every voter who prefers the Condorcet winner to the candidate who would win without it approves the Condorcet winner alone.

The theory rests on three assumptions, which are that voters know who would win each run-off, that a Condorcet winner exists, and that three candidates compete. Relaxing any of them leaves some voters without a dominant ballot, and their best ballots then depend on what they believe about how others will vote. Each relaxation therefore raises a question of strategic voting in the sense of Myerson and Weber (1993), which the theory of large Poisson games can address (Myerson, 1998, 2000). Future work can also model candidates' decisions to run when they are unsure who the Condorcet winner is. The results bear on any future attempt to bring the special elections that fill aldermanic vacancies under AVR. A charter amendment that would have done so won 56 percent of the vote in November 2026, short of

the 60 percent it needed.

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I used Claude (Anthropic), a large language model, to help generate tables and figures. I checked all of its output and take full responsibility for the content.

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